2018

MATHEMATICS – HONOURS

Fourth Paper

(Module - VIII)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer ***any one*** of the following : 3

(a) Find the cartesian coordinates of the point whose spherical polar coordinates are $\left(3, \frac{2\pi}{3}, -\frac{\pi}{6}\right)$.

(b) Show that the quadric surface $2x^2 + 5y^2 + 3z^2 - 4x + 20y - 6z + 5 = 0$ represents an ellipsoid. Find its principal planes. 1+2

2. Answer ***any two*** of the following : 6×2

(a) Show that the equation of the sphere through the circle $x^2 + y^2 + z^2 - 2x - 3y + 4z + 8 = 0 = x^2 + y^2 + z^2 + 4x + 5y - 6z + 2$ and having the centre on the plane $4x - 5y - z = 3$ is $x^2 + y^2 + z^2 + 7x + 9y - 11z = 1$.

(b) (i) Find the equation of the right circular cylinder whose guiding curve is the circle through (1, 0, 0), (0, 1, 0) and (0, 0, 1). 3

(ii) Find the equation of the right circular cone having vertex at origin and axis making equal angles with the coordinate axes, given that the generator has the direction ratios 1, –2, 2. 3

(c) Show that the hyperbolid of one sheet is the only ruled central conicoid.

(d) Establish that the necessary and sufficient condition for the plane $lx + my + nz = p$ to be a tangent plane to the paraboloid $ax^2 + by^2 = 2cz$ is $\frac{l^2}{a} + \frac{m^2}{b} + \frac{2np}{c} = 0$.

Group - B

3. Answer ***any one*** of the following : 10

(a) (i) A system of coplanar forces has the total moments H, $2H$ respectively about points whose coordinates are $(2a, 0)$, $(0, a)$ referred to a fixed rectangular axes. The total resolved parts of the forces along the line $y = x$ vanishes. Find the points in which the line of action of the resultant meets the coordinate axes. 6

(ii) Explain with figure 'the angle of friction' and 'the cone of friction'. 2+2

(b) (i) A particle is resting in equilibrium on a curve at P under the action of forces $\frac{\mu}{r}$ and $\frac{\mu}{r'}$ towards two fixed points O and O' respectively, such that $OP = r$ and $O'P = r'$. Find the locus of P. 5

(ii) A particle rests on a rough curve whose equation is $f(x, y) = 0$ and is acted on by forces, the sums of whose components along the axes of x and y are X and Y respectively. Prove that the particle will rest at all points on the curve at which $\frac{Xf_x + Yf_y}{\sqrt{X^2+Y^2}\sqrt{f_x^2+f_y^2}} > \cos\lambda$, where λ is the angle of friction. 5

4. Answer ***any one*** of the following : 7

(a) A ball falls from a height 'h' upon a fixed horizontal plane. Show that the whole distance traversed before the ball finished rebounding is $\frac{1+e^2}{1-e^2} \cdot h$ and the time taken is $\sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$, '$e$' being the coefficient of restitution. 4+3

(b) A uniform chain of length '$2a$' is hung over a smooth peg so that the length of it on two sides are '$a + b$' and '$a - b$'. If motion starts at this point of time, find the time when the chain leaves the peg.

5. Answer ***any two*** of the following : 9×2

(a) (i) Find the tangential and the normal components of acceleration of a particle in a plane curve. 6

(ii) Prove that for a particle moving down a smooth inclined plane, the sum of kinetic energy and the potential energy is constant. 3

(b) A particle of unit mass is projected with velocity 'u' at an angle 'α' with the horizon in a medium, the resistance of which is K times the velocity. Show that its direction will make an angle '$\frac{\alpha}{2}$' with the horizon after a time $\frac{1}{K}\log\left(1+\frac{Ku}{g}\tan\frac{\alpha}{2}\right)$ and angle 'α' with the horizon after a time $\frac{1}{K}\log\left(1+\frac{2Ku}{g}\sin\alpha\right)$. 5+4

(c) (i) A particle moves in a straight line with acceleration $n^2 \times$ (distance) towards a fixed point in the line and with an additional periodic push 'L Cos pt' [L, n, p are constants]. If the particle starts from rest at a distance 'a' from the centre, find its distance from the fixed point in terms of time. 6

(ii) If in a motion of two dimensions, radial and cross-radial components of velocity are equal, find the equation of the path. 3

(d) (i) A particle is at rest in equilibrium under the attraction of two centres of force. The forces attract directly as the distances and their attractions per unit mass at unit distance being λ and λ'. The particle is now slightly displaced towards one of the centres. Prove that the time of small oscillation is $\frac{2\Pi}{\sqrt{\lambda+\lambda'}}$. 5

(ii) Explain 'terminal velocity' for the motion of a particle in a resisting medium. If a particle falls vertically from rest in a medium whose resistance varies as the 'n' – th power of the velocity, then find the expression for the terminal velocity of the particle, where the acceleration due to gravity is assumed to be constant and 'n' a positive integer. 2+2
